A PHYSICS-INFORMED NEURAL NETWORK FRAMEWORK FOR LINEAR ELASTIC ANALYSIS: FORWARD SIMULATION, PARAMETRIC SURROGATE MODELING, AND INVERSE IDENTIFICATION OF A DEEP TUNNEL

APISIT ROBJANGHVAD, PHAKHIN KHAOWONG, SIRIPORN PANTAN, PANATOUCH THONG-OU

Geotechnical Engineering, Faculty of Engineering, King Mong's University of Thonburi, Bangkok, Thailand

1. Introduction

Solid-mechanics modelling lies on a spectrum from pure physics (FEM) to pure data (ANNs). PINNs sit in between embedding PDEs and boundary conditions into the network's loss function. This enables mesh-free, low-data, and inverse-capable simulation of linear elastic materials.

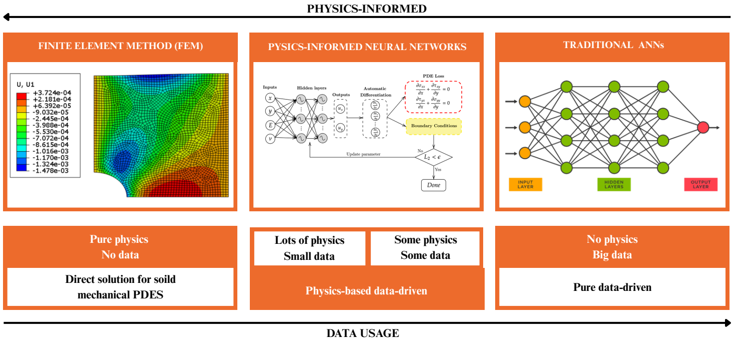


FIG. 1 SPECTRUM OF SOLID MECHANICS MODELING APPROACHES

2. Objective

1.To develop a framework of Physics-Informed Neural Network (PINN) for linear elastic materials.
2.To develop a PINN-based surrogate model and Compare it with FEM.
3.To identify materials properties from inverse analysis.

3. Methodology

Model Architecture

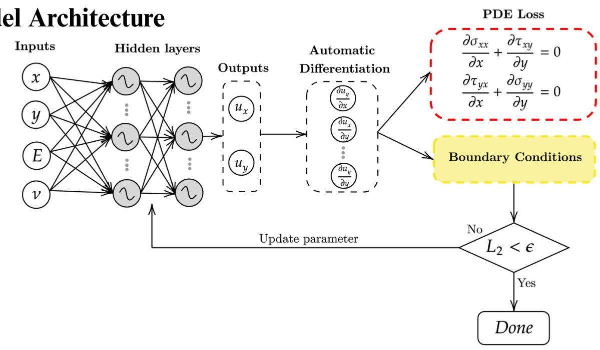


FIG. 2 PINN MODEL ARCHITECTURE

Problem statement

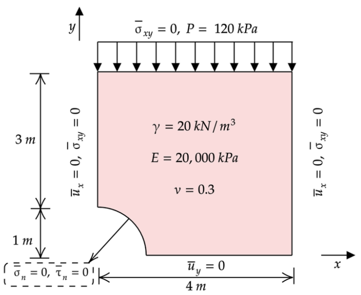


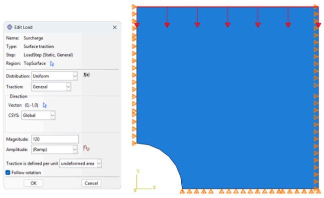
FIG. 3 PROBLEM STATEMENT

Domain: 4x4 square, plane strain conditions.
Geometry: A quarter perforated hole (1.0 m radius) at the lower left.
Loading: Surface compression pressure to the top boundary 120 kPa.
Material Model: Linear Elastic

Finite Element Method (FEM)

Material Model: Linear Elastic

- Young's Modulus = 20000 kPa
- Poisson's ratio = 0.3



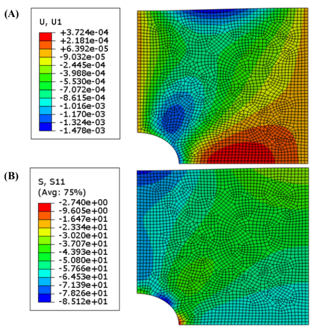


FIG. 4 FINITE ELEMENT METHOD (FEM) REFERENCE SOLUTION SHOWING (A) DISPLACEMENT MAGNITUDE U AND (B) STRESS S11 DISTRIBUTIONS FROM ABAQUS

4. Results

Relative Error $$e_r^{(k)}(\mathbf{x}_i) = \frac{|u_k^{pred}(\mathbf{x}_i) - u_k^{ref}(\mathbf{x}_i)|}{max|u_k^{ref}(\mathbf{x}_i)|}$$

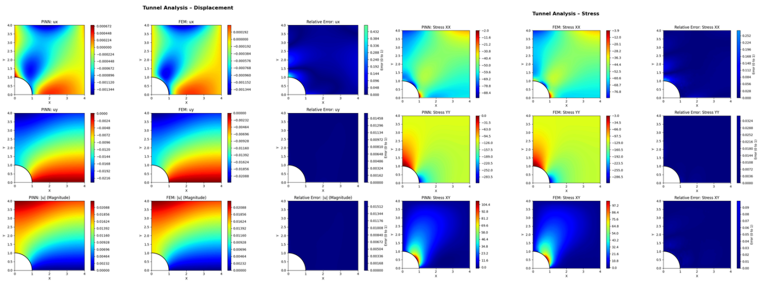


FIG. 5 PINN PREDICTIONS OF DISPLACEMENT AND STRESS FIELDS

Loss

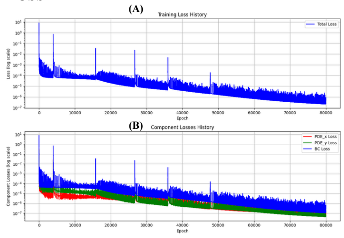


FIG. 6 TRAINING LOSS HISTORY SHOWING (A)TRAINING LOSS HISTORY AND (B) COMPONENT LOSS HISTORY

Parametric Surrogate

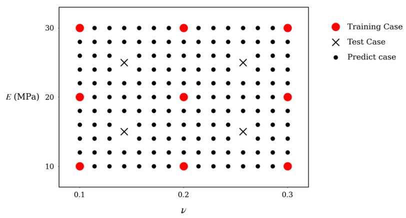


FIG. 7 PARAMETRIC SURROGATE MODEL

Result at test case

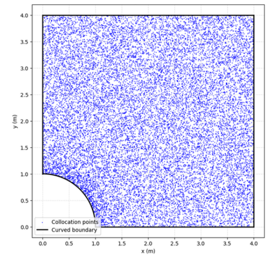


FIG. 8 TEST CASE RESULT SHOWING COLLOCATION POINTS DISTRIBUTION OVER THE COMPUTATIONAL DOMAIN

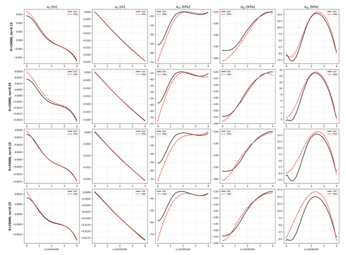


FIG. 9 PINN PREDICTIONS VERSUS FEM REFERENCE ALONG SELECTED SECTIONS FOR DISPLACEMENTS AND STRESSES AT THE TEST CASE

Inverse Analysis

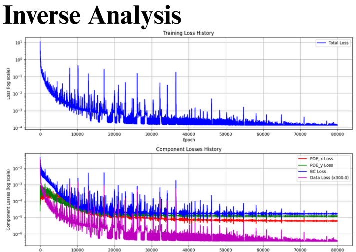


FIG. 10 INVERSE ANALYSIS TRAINING HISTORY

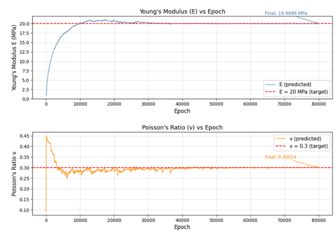


FIG. 11 CONVERGENCE OF IDENTIFIED MATERIAL PARAMETERS DURING INVERSE ANALYSIS

Material Properties	Groundtruth	PINN	Relative Error (%)
Young's Modulus (E) MPa	20	19.9896	0.052
Poisson's ratio (ν)	0.3	0.300024	0.080

TABLE 1: COMPARISON OF IDENTIFIED MATERIAL PROPERTIES FROM INVERSE ANALYSIS VERSUS GROUND TRUTH VALUES

5. Conclusion

1. **PINN Framework Developed**
   - 1.1 PINN framework for **linear elastic materials**
   - 1.2 Physics embedded in loss function
   - 1.3 **Mesh-free** displacement & stress prediction
2. **PINN Surrogate vs FEM**
   - 2.1 **Parametric surrogate** for varying E and ν
   - 2.2 Strong agreement with Abaqus FEM
   - 2.3 Accurate ux , uy , σ_x , σ_y, τ_{xy} predictions
3. **Inverse Analysis Success**
   - 3.1 E = 19.99 MPa (error 0.052%)
   - 3.2 ν = 0.3002 (error 0.080%)
   - 3.3 PINN native capability for **inverse problems**

6. Reference

Github Application Canva Youtube